characteristic Impedance of Transmission line

$$\frac{dv}{dx} = -(R + j\omega L) I$$

$$\frac{d}{dx}\left(V^+ e^{-\gamma x} + V^- e^{+\gamma x}\right) = -(R + j\omega L)\left\{I^+ e^{-\gamma x} + I^- e^{+\gamma x}\right\}$$

$$\underbrace{V^+ e^{-\gamma x} \times -\gamma}_{\text{wave travelling forward}} + \underbrace{V^- e^{\gamma x} \times \gamma}_{\text{wave travelling backward}} = -(R + j\omega L)\left\{I^+ e^{-\gamma x} + I^- e^{+\gamma x}\right\}$$

wave travelling ~~forward~~ forward ~~backward~~

wave travelling ~~forward~~ backward.

so

$$V^+ e^{-\gamma x} \times -\gamma = -(R + j\omega L) I^+ e^{-\gamma x}$$

so $$\frac{V^+}{I^+} = \frac{\cancel{-}(R + j\omega L)\cancel{e^{-\gamma x}}}{\cancel{e^{-\gamma x}} \times \cancel{-}\gamma}$$

$$\frac{V^+}{I^+} = \left(\frac{R + j\omega L}{\gamma}\right).$$

$$\therefore \gamma^2 = (R + j\omega L)(G + j\omega C).$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{R + j\omega L} \times \sqrt{G + j\omega C}$$

$$\frac{V^+}{I^+} = \frac{R + j\omega L}{\sqrt{R + j\omega L} \times \sqrt{G + j\omega C}}$$

or $$\frac{V^+}{I^+} = \frac{\cancel{\sqrt{R+j\omega L}} \times \sqrt{R+j\omega L}}{\cancel{\sqrt{R+j\omega L}} \times \sqrt{G+j\omega C}}$$

$$\boxed{\frac{V^+}{I^+} = \frac{\sqrt{R+j\omega L}}{\sqrt{G+j\omega C}} = Z_0.}$$

Z_0 = characteristic Impedance.
Because it consist of all primary constant.

for wave travelling in -ve direction or backward direction

$$V^- e^{\gamma x} \gamma = -(R+j\omega L) I^- e^{-\gamma x}$$

$$\frac{V^-}{I^-} = \frac{-(R+j\omega L)}{\gamma}$$

$$\boxed{Z_0^- = \frac{V^-}{I^-} = -\sqrt{\frac{R+j\omega L}{G+j\omega C}}.}$$

Z_0^- = -ve characteristic Impedance of Transmission line.